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The Olympic 500-m speed skating; the inner–outer lane difference

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In 1998, the International Skating Union and the International Olympic Committee decided to skate the 500-m twice during World Single Distances Championships, Olympic Games, and World Cups. The decision was based on a study by the Norwegian statistician N. L. Hjort, who showed that in the period 1984–1994, there was a significant difference between 500-m times skated with a start in the inner and outer lanes. Since the introduction of the clap skate in the season 1997–1998, however, there has been a general feeling that this difference is no longer significant. In this article we show that this is, in fact, the case.

Keywords and Phrases: sports, clap skate, statistics.

1 Inner–outer lane problem

A 500-m speed skating race is performed by pairs of skaters. A draw decides whether a skater starts in either the inner or the outer lane of the 400-m rink; see Figure 1.

After the first curve, skaters change lanes at the ‘crossing line’. The second curve is the major cause of the difference in time between inner and outer starts. Since the inner lane is tighter compared with the outer lane, skaters reach a higher acceleration force there. So skaters who finish in the inner lane seem to have a disadvantage owing to control problems. From the 500-m results of the World Sprint Championships between 1984 and 1994, Hjort (1994) showed that this disadvantage indeed exists. There is a general feeling, however, that this disadvantage has declined since the introduction of the clap skate in the season 1997–1998; clap skates seem to

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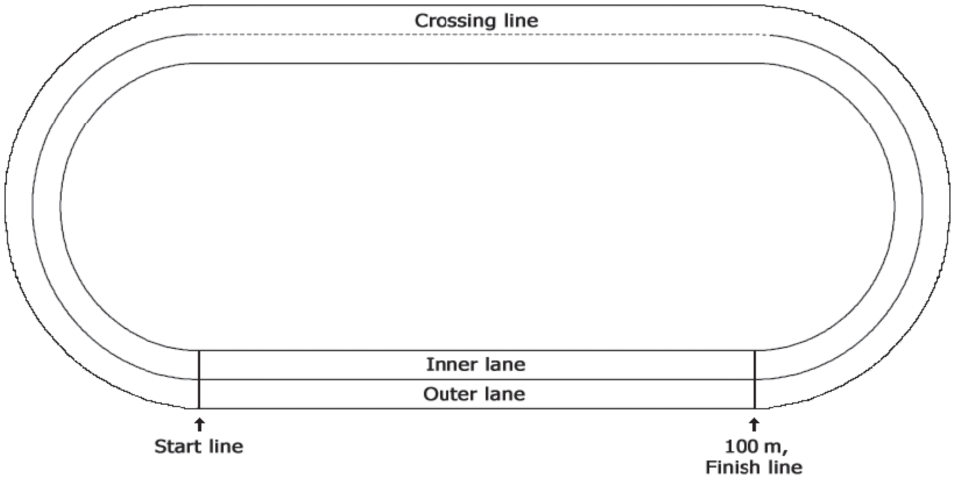

Fig. 1. Schematic speed skating rink.

provide more grip in the curves. In this article, we show that the disadvantage of the last inner lane no longer exists in terms of statistical significance. Our data set includes starting lists of the 500-m men's races of the World Sprint Championships, the World Single Distances Championships, and eleven World Cup events between 2002 and 2008.

2 The statistical model

Indices and parameters describing clock times for the first 100-m (also called the 100-m splitting times), and the finishing times are as follows:

- k Tournament index; with 23 tournaments in the data set, $k \in \{1, \dots, 23\}$;
- N_k The set of skaters who participated in tournament k;
- n_k The number of participants in tournament k, $n_k = |N_k|$;
- N_k^I The set of skaters who started in the inner lane in race 1 on tournament k;
- N_k^O The set of skaters who started in the outer lane in race 1 on tournament k;
- i Skater index; skater $i \in N_k$ refers to the ith skater on the starting list of tournament k;
- h Race index; $h = 1, 2$;
- X_{hik} The 100-m splitting time of skater $i \in N_k$ in race h on tournament k;
- Y_{hik} The 500-m finishing time of skater $i \in N_k$ in race h on tournament k;
- C_{ik} Random effect parameter;

$$W_{ik} = \begin{cases} -1/2 & \text{if } i \in N_k^I \\ +1/2 & \text{if } i \in N_k^O. \end{cases}$$

The last parameter reflects the inner–outer information. Let $i \in N_k$ for $k = 1, \dots, 23$. Note that $N_k = N_k^I \cup N_k^O$. The parameter C_{ik} models the 'inverse' ability of skater

$i \in N_k$ compared with an average skater, meaning that low/high values of C_{ik} correspond to better/worse performances, respectively. The value of C_{ik} cannot be estimated directly, but is assumed to satisfy $C_{ik} \sim N(0, \kappa_k^2)$. As in Hjort (1994), the 500-m time is modeled as the sum of a constant, a multiple of the 100-m time, a measure of the skater's ability, the inner–outer information, and a random term:

$$Y_{hik} = a_{hk} + b_{hk} X_{hik} + C_{ik} + d_k W_{ik} + \epsilon_{hik} \tag{1}$$

with $h = 1, 2$, and a_{hk} is the constant that measures the gliding and weather conditions during race h on tournament k; b_{hk} is the coefficient that corresponds to the 100-m time skated and that takes the gliding and weather conditions into account of race h on tournament k; d_k is the parameter that measures the difference between the 500-m times skated with a start in the inner and outer lanes on tournament k; ϵ_{hik} is the random discrepancy between the actual and estimated 500-m times of skater i in race h on tournament k.

The parameter ϵ_{hik} is also called the error term. It is assumed that ϵ_{1ik} and ϵ_{2ik} are independent, and that they satisfy $\epsilon_{hik} \sim N(0, \sigma_{hk}^2)$. For each tournament, Bartlett's test (see Miller and Miller, 2004) is used to justify the assumption of constant variability, or $\sigma_{1k}^2 = \sigma_{2k}^2 = \sigma^2$ for all k. The estimated 500-m time of skater i in race h on tournament k is defined as:

$$\widehat{Y}_{hik} = \widehat{a}_{hk} + \widehat{b}_{hk} X_{hik} + \widehat{d}_k W_{ik}, \tag{2}$$

where a 'hat' means the corresponding estimated value. Assuming that the random effect and the error terms are normally distributed, Model (1) can now be formulated as:

$$\begin{pmatrix} Y_{1ik} \\ Y_{2ik} \end{pmatrix} \sim N_2 \left\{ \begin{pmatrix} a_{1k} + b_{1k} X_{1ik} + d_k W_{ik} \\ a_{2k} + b_{2k} X_{2ik} - d_k W_{ik} \end{pmatrix}, \begin{pmatrix} \sigma_{1k}^2 + \kappa_k^2 & \kappa_k^2 \\ \kappa_k^2 & \sigma_{2k}^2 + \kappa_k^2 \end{pmatrix} \right\}. \tag{3}$$

Note that $\mathrm{var}(Y_{hik}) = \sigma_{hk}^2 + \kappa_k^2$. Since $E(Y_{1ik} Y_{2ik}) = E(Y_{1ik}) E(Y_{2ki}) + \kappa_k^2$, we have that $\mathrm{cov}(Y_{1ik}, Y_{2ik}) = \kappa_k^2$. For every k, we assume that $b_{1k} = b_{2k} = b_k$, meaning that a difference in weather and gliding conditions between first and second races can be completely explained from the difference between a_{1k} and a_{2k}. Generalized likelihood ratio tests (see, e.g., Rice, 1995) on the data support this simplification. One might argue that the assumption that the difference between the finishing times of one skater skated during tournament k can only be attributed to the difference between a_{1k} and a_{2k} is rather strong. However, as data support this assumption, we simplified the model accordingly.

Figure 2 plots and regresses the 100-m splitting times against the 500-m finishing times of all skaters for the first (left) and second (right) races of the World Sprint Championships 2008 in Heerenveen.

Figure 3 displays the relationship between the 100-m splitting times and the residuals (the difference of the actual and fitted 500-m times) for the World Sprint Championships 2008 in Heerenveen.

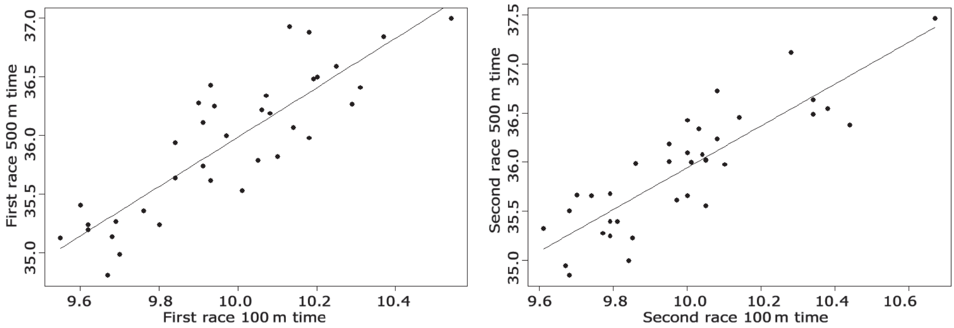

Fig. 2. Relationship between the 100- and 500-m times of the World Sprint Championships 2008 in Heerenveen.

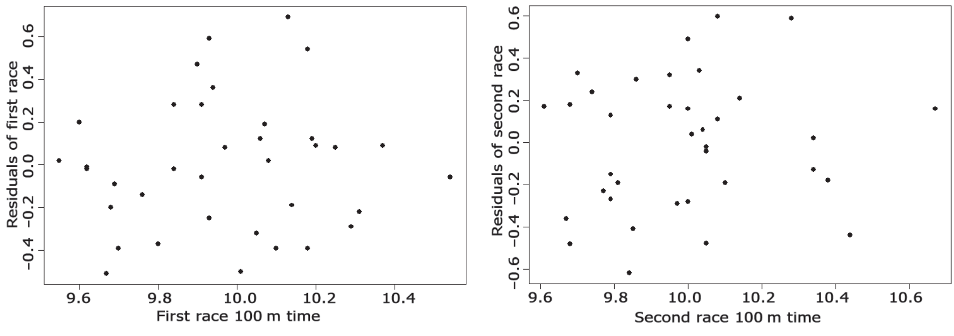

Fig. 3. Relationship between the 100-m times and the residuals of the World Sprint Championships 2008 in Heerenveen.

The left and right panels of Figure 3 plot the 100-m splitting times against the residuals for the first and second races, respectively. For example, during the second race there is a skater with a 100-m splitting time of 10.1 and a difference of approximately -0.2 seconds between the actual and estimated 500-m times. The means of the residuals are -0.005 and -0.004 for the first and second races, respectively, which shows that the error terms have a zero mean. The variances of the residuals are 0.093 and 0.097 for the first and second races, respectively, leading to the conclusion that the error terms have an equal variability.

For every k, the correlation coefficient between Y_{1ik} and Y_{2ik}, denoted by ρ_k, satisfies

$$\rho_k = \frac{\operatorname{cov}\left(Y_{1ik}\, Y_{2ik}\right)}{\sqrt{\operatorname{var}\left(Y_{1ik}\right) \operatorname{var}\left(Y_{2ik}\right)}} = \frac{\kappa_k^2}{\sigma_k^2 + \kappa_k^2}.$$

This correlation coefficient measures the strength of the relationship between the 500-m times of the first and second races. It is expected that ρ_k is close to 1, since, for each skater, there is a strong positive dependency between the performances of both races. In other words, ρ_k measures the performance stability of an average skater during tournament k.

Table 1. Results of the first eight skaters of the World Single Distances Championships 2008 in Nagano. (I, start in the inner lane; O, start in the outer lane)

			First race			Second race	
			100-m	500-m		100-m	500-m
1	Jeremy Wotherspoon	I	9.69	34.78	O	9.60	34.68
2	Kyou-Hyuk Lee	I	9.71	35.11	O	9.60	34.90
3	Joon Mun	I	9.88	35.14	O	9.92	35.21
4	Dmitry Lobkov	O	9.73	35.21	I	9.75	35.17
5	Joji Kato	O	9.68	35.25	I	9.65	35.07
6	Keiichiro Nagashima	O	9.77	35.35	I	9.69	35.10
7	Mika Poutala	I	9.68	35.35	O	9.70	35.26
8	Mike Ireland	I	9.86	35.44	O	9.85	35.21

To estimate the model, we use the 100- and 500-m times of both races, as well as the starting position of the skaters. Table 1 presents the values of the afore described parameters for the first eight skaters from the starting list of the World Single Distances Championships 2008 in Nagano.

Model 3 is estimated by using the mixed model functions in S-PLUS. The estimates of σ_k and ρ_k are adjusted for the sample size of tournament k, leading to the sample size-corrected maximum likelihood estimator for σ_k^2, denoted and defined by

$$\widehat{\sigma}^2_{un,k} = \frac{2n_k}{2n_k - p}\hat{\sigma}^2_k.$$

It follows from the log-likelihood function that the parameter κ_k is not estimated directly, so that p, the number of estimated parameters, is 6. The corrected estimator of κ_k^2 is denoted and defined by

$$\widehat{\kappa}^2_{un,k} = \hat{\sigma}^2_{un,k}\frac{\hat{\rho}_k}{1-\hat{\rho}_k}.$$

3 Outliers

Speed skating is a technical sport that includes falls and minor slips, leading to results that deviate from average performances. We select skaters with two error-free races by eliminating skaters with at least one race that contains a fall or a slip. To achieve this, the procedure of detecting outliers from Hjort (1994) is used.

In Hjort (1994), three test statistics are specified of which the first two, denoted by $T_{1k}(i)$ and $T_{2k}(i)$, test the first and second 500-m time of skater $i \in N_k$ skated during tournament k, respectively. According to Hjort (1994), skater i is defined to be an outlier if the value of either $T_{1k}(i)$ or $T_{2k}(i)$ exceeds 2.75. The third test statistic, denoted by $T_{3k}(i)$, tests the difference between lap times, where the lap time of skater $i \in N_k$ in race h is defined as: $R_{hik} = Y_{hik} - X_{hik}$. Again similar to Hjort (1994), skater i is an outlier if $|T_{3k}(i)| \geq 2.75$.

We explain the procedure of detecting outliers by using results of the 2006 World Sprint Championships in Heerenveen (tournament index 8). The results of Wotherspoonare:

		First race			Second race	
		100-m	500-m		100-m	500-m
Jeremy Wotherspoon	I	11.84	45.73	O	9.83	35.47

Since $R_{2,8}$(Wotherspoon) $-$ $R_{1,8}$(Wotherspoon) $= -8.25$, he is an outlier: his first race was not error-free. After Wortherspoon is removed from the starting list, Model (3) is estimated, with $\widehat{a}_1 = 5.441$, $\widehat{a}_2 = 5.508$, $\widehat{b} = 3.071$, $\widehat{d} = 0.062$, $\widehat{\rho} = 0.774$, $\widehat{\sigma} = 0.165$, $\widehat{\kappa} = 0.305$. Using Equation (2) and these estimated parameters, we predict that Ustynowicz skates $(5.508 + 3.071 \cdot 9.83 - (-0.5) \cdot 0.062 =)$ 35.73 in his second race. There is a difference of one second between the expected and the actual 500-m time of his second race, because his results are:

		First race			Second race	
		100-m	500-m		100-m	500-m
Maciej Ustynowicz	I	9.86	36.31	O	9.83	36.73

It follows that $\widehat{T}_{3,8}$(Ustynowicz) $= 2.798$, meaning that Ustynowicz is indeed an outlier. After removing him from the starting list, the estimated parameters satisfy $\widehat{a}_1 = 4.861$, $\widehat{a}_2 = 4.919$, $\widehat{b} = 3.127$, $\widehat{d} = 0.054$, $\widehat{\rho} = 0.749$, $\widehat{\sigma} = 0.163$, and $\widehat{\kappa} = 0.282$. Similarly, substituting these values in the test statistics yields $\widehat{T}_{3,8}$(Bélanger) = 2.783 and $\widehat{T}_{3,8}$(Vtipil) $= 2.863$, which are also outlier values.

4 Analysis of the data between 1984–1994 and 2002–2008

First, we estimate Model (3) with the data from the period 1984–1994, followed by applying the three outlier tests. Only skaters who pass the three outlier tests are used. We observe the following deviations from the findings of Hjort (1994). In our analysis of the 1985 World Sprint Championships, Henriksen is not an outlier, since $\widehat{T}_{1,1985}$(Henriksen) $= -1.951$, $\widehat{T}_{2,1985}$(Henriksen) $= -3.147$, and $\widehat{T}_{3,1985}$(Henriksen) $= -1.383$, whereas he is in Hjort (1994). Similarly, Konincx is not an outlier in 1994, since $\widehat{T}_{1,1994}$(Konincx) $= 2.413$, $\widehat{T}_{2,1994}$(Konincx) $= 2.488$, and $\widehat{T}_{3,1994}$(Konincx) $= 0.154$. The overall estimator of d between 1984–1994 is denoted by $\widehat{d}_{1984-1994}$ (for definition, see Hjort 1994). The standard error of the estimator decreases as the number of tournaments analyzed increases. According to Hjort (1994), the standard deviation of $\widehat{d}_{1984-1994}$ is 0.02 when 300 skaters' paired runs are analyzed, corresponding to eleven World Sprint Championships. The variance of the overall estimate of d is denoted by var $\hat{d}_{1984-1994}$ (for definition, see Hjort, 1994). It follows that the estimate of $\widehat{d}_{1984-1994}$ equals 0.049 with a standard deviation of 0.015, but according to Hjort (1994), the estimate and standard error are 0.048 and 0.016, respectively.

We have tested the null hypothesis $[\mathrm{H}_0 : d_{1984-1994} = 0]$ against the alternative hypothesis $[\mathrm{H}_A : d_{1984-1994} \neq 0]$. Using the test statistic

$$T = \frac{\widehat{d}_{1984-1994}}{\sqrt{\operatorname{var} \widehat{d}_{1984-1994}}}, \tag{4}$$

which follows a standard normal distribution under H_0, the result is $T=0.049/0.015=3.267$, leading to a P-value of 0.001. At a 5% significance level we agree with Hjort (1994) that the null hypothesis should be rejected in the period 1984–1994. Hence, a skater finishing in the outer lane has a significant advantage over his rival finishing in the inner lane in this period.

We now show that for the clap skate period, the hypothesis cannot be rejected anymore. To that end, we seek an estimate of $d_{2002-2008}$ using paired runs of 300 skaters. Data from fourteen tournaments after 2002 is used, allowing six years for skaters to become comfortable with the clap skate (introduced in the season 1997–1998). The tournaments analyzed, together with the estimates of the seven parameters and the standard deviation of $\widehat{d}$, are presented in Table 2.

Based on the values of Table 2, we have that the estimated value and the standard error of $\widehat{d}_{2002-2008}$ are 0.006 and 0.010, respectively. From Equation (4), it follows that $T=0.006/0.010=0.6$. The P-value, corresponding to the test of the null hypothesis $[H_0 : d_{2002-2008}=0]$ against the alternative $[H_A : d_{2002-2008} \neq 0]$ is 0.548, leading to the conclusion that we do not reject the null hypothesis at a significance level of 5%. Even when we test the null hypothesis against its one-sided alternative $[H_A : d_{2002-2008} > 0]$, the null hypothesis is not rejected, since the latter test has a P-value of 0.274. The 95% confidence interval for $d_{2002-2008}$ is $(-0.014; 0.026)$. Estimates of d with a low standard error, estimate d more precisely compared with those with having a high standard error. We observe that the estimates of d of the World Single Distances 2003, 2004, and the 2007/2008 World Cup events have the highest standard error, namely 0.045. Removing the four estimates, such as to find out whether they influence our inferences, leads to an estimate of 0.001 with a corresponding standard error of 0.011. Testing $[H_0 : d_{2002-2008}=0]$ against the alternative $[H_A : d_{2002-2008} \neq 0]$,

Table 2. Estimated parameters for the tournaments between 2002 and 2008. (WSC, World Sprint Championships; WSDC, World Single Distances Championships; and WC, World Cup)

Year	Place	Event	$\widehat{a}_1$	$\widehat{a}_2$	$\widehat{b}$	$\widehat{d}$	$se_{\widehat{d}}$	$\widehat{\rho}$	$\widehat{\sigma}$	$\widehat{\kappa}$
2002	Hamar	WSC	11.659	11.665	2.463	−0.022	0.035	0.806	0.142	0.289
2003	Calgary	WSC	16.250	16.195	1.964	−0.054	0.041	0.857	0.165	0.404
2003	Berlin	WSDC	21.118	21.049	1.509	0.047	0.045	0.876	0.145	0.385
2004	Nagano	WSC	14.178	14.192	2.199	−0.011	0.041	0.796	0.151	0.298
2004	Seoul	WSDC	16.321	16.237	1.997	0.046	0.045	0.783	0.134	0.255
2005	Salt Lake City	WSC	15.718	15.683	1.987	−0.007	0.037	0.828	0.131	0.287
2005	Inzell	WSDC	23.094	23.018	1.329	−0.021	0.037	0.823	0.121	0.261
2006	Heerenveen	WSC	4.890	4.916	3.125	0.022	0.027	0.839	0.130	0.297
2007	Hamar	WSC	18.335	18.375	1.736	0.042	0.040	0.683	0.142	0.208
2007	Salt Lake City	WSDC	19.218	19.246	1.601	0.021	0.042	0.695	0.145	0.219
2007	Salt Lake City	WC	19.337	19.292	1.600	0.024	0.045	0.745	0.163	0.279
2007	Calgary	WC	26.086	26.176	0.879	−0.012	0.045	0.713	0.134	0.211
2008	Heerenveen	WSC	16.004	15.965	1.999	0.031	0.031	0.819	0.130	0.277
2008	Nagano	WSDC	12.695	12.710	2.325	−0.022	0.036	0.824	0.115	0.249

yields a test statistic of 0.091. The corresponding P-value is 0.928, and the null hypothesis is not rejected. Hence, the evidence against the null hypothesis is diminished when unsecure esimates are removed. The 95% confidence interval of $d_{2002-2008}$ increases to (−0.021; 0.023).

As skaters are faster on relatively high altitude rinks, they experience a higher acceleration force in the inner lane. Therefore, the advantage of a finish in the outer lane may persist at high altitude rinks. Accordingly, we analyze the World Cup events from three low and two high altitude rinks (Table 3).

Removing data of World Cup events in the period 2002–2008 that do not satisfy the assumptions of Model (3), we remain with 260 skaters' paired runs. Based on the values of d for the World Single Distances Championships, the World Sprint Championships, and the World Cups, we estimate dl and dh, denoting the average difference between the 500-m times skated with a start in the inner and outer tracks for the low and high altitude rinks, respectively.

The seasons and the locations where the World Cups are organized are listed in Table 4, as well as the estimates of the parameters, and the standard deviation values of $\widehat{d}$.

The statistics in Table 5 are obtained by substituting the results of Table 4 into the definitions and Equation (4).

To determine whether or not the inferences are influenced by the estimates of d with a high standard error, we exclude the estimates of d for the World Cups organized in Heerenveen in the seasons 2004–2005 and 2007–2008, and of the World Single Distances Championships 2003 in Berlin for the low altitude rinks. For the high altitude rinks, the estimates of d for the World Cups organized in Calgary in the seasons 2004–2005 and 2007–2008 are removed, together with the estimate of d

Table 3. Height above sea level.

Location	Altitude (m)
Heerenveen	0.4
Berlin	34
Hamar	125
Calgary	1034
Salt Lake City	1425

Table 4. Estimated parameters for World Cups between 2002 and 2008

Season	Location	$\widehat{a}_1$	$\widehat{a}_2$	$\widehat{b}$	$\widehat{d}$	$se_{\widehat{d}}$	$\widehat{\rho}$	$\widehat{\sigma}$	$\widehat{\kappa}$
2004–2005	Heerenveen	14.840	14.837	2.123	−0.021	0.051	0.810	0.134	0.277
2006–2007	Heerenveen	17.833	17.819	1.822	−0.052	0.043	0.700	0.152	0.232
2007–2008	Heerenveen	24.372	24.377	1.121	−0.042	0.045	0.691	0.133	0.199
2007–2008	Hamar	19.659	19.615	1.585	0.025	0.026	0.927	0.080	0.285
2006–2007	Berlin	21.432	21.297	1.442	0.012	0.042	0.574	0.131	0.152
2003–2004	Salt Lake	22.135	22.114	1.331	0.075	0.038	0.788	0.121	0.233
2005–2006	Salt Lake	4.801	4.761	3.100	0.038	0.033	0.850	0.155	0.369
2003–2004	Calgary	4.649	4.625	3.139	0.005	0.040	0.814	0.194	0.406
2004–2005	Calgary	21.063	21.040	1.454	−0.024	0.046	0.635	0.145	0.191

Table 5. Estimated parameters, test statistics and P-values for low and high altitude rinks

Number of skaters' paired runs	$\widehat{dl}$	$se_{\widehat{dl}}$	T	Two-sided P-value	One-sided P-value
259	0.010	0.011	0.909	0.364	0.182
204	0.011	0.013	0.846	0.398	0.199
Number of skaters' paired runs	$\widehat{dh}$	$se_{\widehat{dh}}$	T	Two-sided P-value	One-sided P-value
257	0.011	0.013	0.846	0.398	0.199
191	0.016	0.016	1	0.318	0.159

for the World Cup organized in Salt Lake City in the season 2007–2008. These estimates of dl and dh are given in the second and fourth lines of Table 5, respectively. For the low altitude rinks, Table 5 gives a two- and one-sided P-value which are a result of the test $[\mathrm{H}_0: dl=0]$ against $[\mathrm{H}_A: dl \neq 0]$, and of $[\mathrm{H}_0: dl=0]$ against $[\mathrm{H}_A: dl>0]$, respectively. The two- and one-sided P-values are obtained in a similar way for the high altitude rinks. Clearly, the four estimates of d are not significant at a significance level of 5%, indicating that there is no significant difference between the 500-m times skated with a start in the inner and outer lanes on low and high altitude rinks.

Turning back to the original question, namely whether d is influenced by altitude, the hypotheses reads $[\mathrm{H}_0: dh=dl]$ against $[\mathrm{H}_A: dh>dl]$. The test is performed with the statistic

$$T\left(dh-dl\right)=\frac{(\widehat{dh}-\widehat{dl})-\left(dh-dl\right)}{s_p\sqrt{\frac{1}{nh}+\frac{1}{nl}}},$$

which contains the pooled sample variance s_p with

$$s_p^2=\frac{\left(nh-1\right)se_{dh}^2+\left(nl-1\right)se_{dl}^2}{nh+nl-2}.$$

Using the estimates of Table 5, $nh=257$ and $nl=259$, yielding that $s_p=0.012$. Hence,

$$T\left(dh-dl\right)=\frac{\left(0.011-0.010\right)-\left(dh-dl\right)}{0.012\sqrt{\frac{1}{257}+\frac{1}{259}}}.$$

For the null hypothesis, we have that

$$T(0)=\frac{\left(0.011-0.010\right)}{0.012\sqrt{\frac{1}{257}+\frac{1}{259}}}=0.946,$$

following a t-distribution with $nh+nl-2=257+259-2=514$ degrees of freedom (see Rice 1995). For this one-sided test, the P-value is 0.172, leading to the conclusion that we do not reject the null hypothesis. So, a higher acceleration force in the curve of the inner lane in Calgary and Salt Lake City does not lead to an estimate of d that differs significantly from the low altitude rinks. Hence, finishing in the outer lane at high altitude rinks is not more advantageous to low altitude rinks. Note that a test of the null hypothesis [$\mathrm{H}_0 : dh = dhl$] against the alternative hypothesis [$\mathrm{H}_A : dh > dhl$] yields the same conclusion, since the corresponding P-value is 0.095. Here, $\widehat{dhl}$ is the estimate of d that is based on all data of the low and high altitude rinks.

Finally, we present an estimate of d, denoted by $\widehat{d}_{2002-2008}$, which is based on all data between 2002 and 2008. Substituting all estimates of d into the definitions gives that $\widehat{d}_{2002-2008} = 0.008$ with a standard error of 0.008. To find the value of the test statistic that corresponds to the test of the null hypothesis [$\mathrm{H}_0 : d_{2002-2008} = 0$] against the alternative [$\mathrm{H}_A : d_{2002-2008} \neq 0$], Equation (4) is used, yielding $T = 1$ with a P-value of 0.318. So, at a significance level of 5% we do not reject the null hypothesis. Even when we test the null hypothesis against the one-sided alternative, we do not reject the null hypothesis. The 95% confidence interval for $d_{2002-2008}$ is (-0.008; 0.024). The highest standard errors are observed at the 2004–2005 World Cup events organized in Heerenveen and Calgary, namely 0.051 and 0.046, respectively. Removing these two estimates, yields that $\widehat{d}_{2002-2008} = 0.009$ with a standard error of 0.008. According to Equation (4) we have that $T = 1.125$ with a P-value of 0.260, and the null hypothesis is not rejected. Moreover, the null hypothesis is not rejected when we test against the one-sided alternative. The tests and the confidence interval show that in the period 2002–2008, skaters finishing in the outer lane did not have an advantage over their rivals finishing in the inner lane.

5 What would happen in a reverse draw?

In Hjort (1994) a correction of the 500-m times of the Olympic Games held in Calgary (1988), Albertville (1992), and Lillehamer (1994) is presented. The 500-m times of skaters who started in the inner lane were corrected for their advantage by adding 0.048 to their finish time, whereas 0.048 was subtracted from the 500-m times of skaters starting in the outer lane. Hjort (1994) showed that reversing the draw changes the medal winners. In this section, the 500-m times of the World Single Distances Championships in the period 2002–2008 are corrected in a similar way. In contrast to the period 2002–2008, when the 500-m was skated twice, the 500-m was skated once during 1984–1994. Hence, we have the opportunity to correct the 500-m times of the first and second races. To show that skating only one 500-m competition is sufficient at the World Cup events, the World Single Distances Championships, and the Olympic Games, we assume that the ranking of the first race is the final one (Table 6).

Except for the World Single Distances Championships 2007, the top three finishers are identical for all championships. However, the fifth position changes in the World Single Distances Championships 2003 and 2004. There are nine differences between the realized and reversed rankings (8.0%) for all World Single Distances Championships between 2002 and 2008. Comparing both rankings during the period 1984–1994 yields a difference of 63.4%.

Table 6. Realized and reverse draw rankings of the World Single Distances Championships in the period 2002–2008

Realized ranking				Reverse draw ranking			
World Single Distances Championships, Berlin, March 14, 2003							
1	Jeremy Wotherspoon	O	35.12	1	Jeremy Wotherspoon	I	35.11
2	Hiroyasu Shimizu	I	35.19	2	Hiroyasu Shimizu	O	35.20
3	Erben Wennemars	I	35.53	3	Erben Wennemars	O	35.54
4	Dmitry Lobkov	I	35.63	4	Dmitry Lobkov	O	35.64
5	Gerard van Velde	I	35.67	5	Mike Ireland	I	35.67
World Single Distances Championships, Seoul, March 12, 2004							
1	Mike Ireland	O	35.40	1	Mike Ireland	I	35.39
2	Jeremy Wotherspoon	O	35.54	2	Jeremy Wotherspoon	I	35.53
3	Masaaki Kobayashi	I	35.60	3	Masaaki Kobayashi	O	35.61
4	Dmitry Lobkov	I	35.68	4	Dmitry Lobkov	O	35.69
5	Kip Carpenter	I	35.78	5	Casey Fitzrandolph	I	35.77
5	Casey Fitzrandolph	O	35.78	6	Kip Carpenter	O	35.79
World Single Distances Championships, Inzell, March 4, 2005							
1	Joji Kato	O	35.57	1	Joji Kato	I	35.56
2	Hiroyasu Shimizu	I	35.75	2	Hiroyasu Shimizu	O	35.76
3	Jeremy Wotherspoon	I	35.87	3	Jeremy Wotherspoon	O	35.88
4	Dmitry Lobkov	O	35.90	4	Dmitry Lobkov	I	35.89
5	Yuya Oikawa	I	35.93	5	Yuya Oikawa	O	35.94
World Single Distances Championships, Salt Lake City, March 9, 2007							
1	Dmitri Lobkov	I	34.43	1	Dmitri Lobkov	O	34.44
2	Kang-Seok Lee	I	34.44	2	Kang-Seok Lee	O	34.45
3	Yuya Oikawa	I	34.45	3	Yuya Oikawa	O	34.46
4	Kyou-Hyuk Lee	O	34.47	3	Kyou-Hyuk Lee	I	34.46
5	Tucker Fredricks	O	34.48	5	Tucker Fredricks	I	34.47
World Single Distances Championships, Nagano, March 7, 2008							
1	Jeremy Wotherspoon	I	34.78	1	Jeremy Wotherspoon	O	34.79
2	Kyou-Hyuk Lee	I	35.11	2	Kyou-Hyuk Lee	O	35.12
3	Joon Mun	I	35.14	3	Joon Mun	O	35.15
4	Dmitry Lobkov	O	35.21	4	Dmitry Lobkov	I	35.20
5	Joji Kato	O	35.25	5	Joji Kato	I	35.24

6 Conclusion

The argument that skaters finishing in the outer lane have an advantage over skaters finishing in the inner lane, the basis for the decision to skate the 500-m twice at World Cups, World Single Distances Championships, and Olympic Games, is no longer valid. Table 7 summarizes the average difference between 500-m times skated with a start in the inner and outer lanes, denoted by $\widehat{d}$, for the periods 1984–1994 (Hjort, 1994) and 2002–2008.

Our investigations show that the advantage has shifted from 0.048 seconds between 1984 and 1994, to 0.008 seconds between 2002 and 2008. The advantage disappeared after 2002, certainly caused by the introduction of the clap skate. Moreover, we have shown that even on the two high altitude rinks, in Calgary and Salt Lake City, the difference has disappeared.

Furthermore, we considered the percentage of races won by the inner lane starter for both periods. For this analysis, we removed races in which both skaters have

Table 7. Summary of the results from Hjort (1994) and our findings

Data set	$\widehat{d}$	$se_{\widehat{d}}$	95% confidence interval for d
Hjort (1994)	0.048	0.016	(0.017; 0.079)
Current research	0.008	0.008	(−0.008; 0.024)

the same time. It turns out that the percentage of races won by the inner and outer lane starters are 48.7 and 51.3 in the period 1984–1994, respectively, whereas these figures are 50.2 and 49.8 in the period 2002–2008, respectively. The statistics make clear that, in contrast to the period 1984–1994, the majority of skaters is not faster anymore with a finish in the outer lane in the period 2002–2008. Furthermore, we observe that the percentages are shifted toward a 50–50 proportion after the introduction of the clap skate. Consequently, skaters are on average as fast in the inner lane as in the outer lane.

Of course, all this does not mean that the 500m should not be skated twice anymore. On an individual level the difference still exists. However, instead of taking the sum of the two finishing times we would propose to use the best time for the final ranking, so that a mistake or a fall in one race does not immediately mean being prospectless for the medals.

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